



PRODUCTION ALLOCATION THROUGH NONNEGATIVE MATRIX FACTORIZATION*

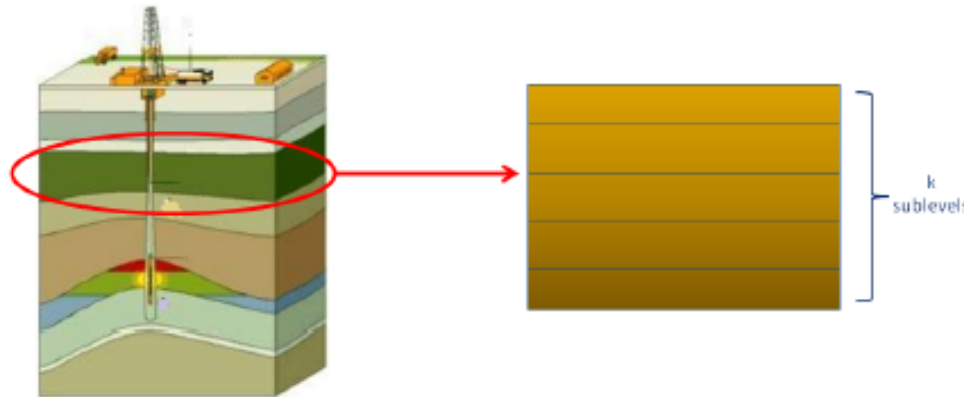
Paolo Zanini

MOX - Department of Mathematics, Politecnico di Milano

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Production Allocation



We consider n wells where oils are extracted from a reservoir with k sublevels.

Starting from the observation of the n chromatograms, we want to retrieve the sublevel chromatograms and the contributions of the sublevels for every well.

For the generic i th well we can consider the following model:

$$X_i = c_{i1} \times S_1 + c_{i2} \times S_2 + \dots$$

The equation is illustrated with chromatograms. On the left is the total chromatogram X_i , which shows two distinct peaks. This is followed by an equals sign and a series of terms. The first term is $c_{i1} \times S_1$, where S_1 is a chromatogram with a single peak. This is followed by a plus sign and the second term $c_{i2} \times S_2$, where S_2 is a chromatogram with a single peak at a different position. The equation ends with a plus sign and an ellipsis, indicating further sublevel contributions.

Nonnegative matrix factorization - Alternating Least Square

$$X = CS$$

X - **data matrix** (# Observations \times # Instants)

Known

C - **concentration matrix** (# Observations \times # Sublevels)

Unknown

S - **sublevel matrix** (# Sublevels \times # Instants)

Unknown

Question

Is it possible to provide and estimate (\hat{C}, \hat{S}) of (C, S) only relying upon data gathered in X ?

Answer: an alternating least square algorithm

$$\hat{C}^{(m)} = \arg \min_C \|X - C \hat{S}^{(m-1)}\|^2$$

$$\hat{S}^{(m)} = \arg \min_S \|X - \hat{C}^{(m)} S\|^2$$

under suitable constraints (nonnegativity, coefficients sum...).

Results

Accuracy: mean error for the coefficient matrix around 3%.