PRODUCTION ALLOCATION THROUGH NONNEGATIVE MATRIX FACTORIZATION*
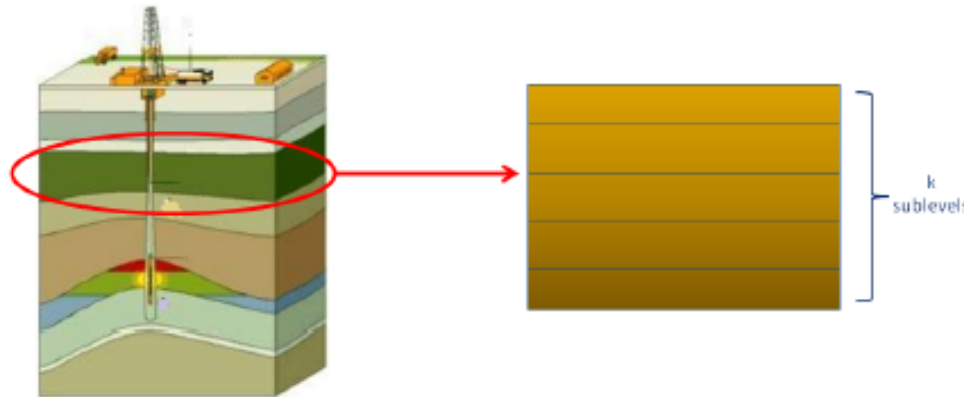
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We consider n wells where oils are extracted from a reservoir with k sublevels.

Starting from the observation of the n chromatograms, we want to retrieve the sublevel chromatograms and the contributions of the sublevels for every well. For the generic ith well we can consider the following model:

$$X_i = c_{i1} \times S_1 + c_{i2} \times S_2 + \cdots$$

Nonnegative matrix factorization - Alternating Least Square

$$X = CS$$

X - **data matrix** (# Observations × # Instants) — Known

C - **concentration matrix** (# Observations × # Sublevels) — Unknown

S - **sublevel matrix** (# Sublevels × # Instants) — Unknown

Question

Is it possible to provide and estimate $(\widehat{C}, \widehat{S})$ of (C, S) only relying upon data gathered in X?

Answer: an alternating least square algorithm

$$\widehat{C}^{(m)} = \arg \min_C \|X - C\widehat{S}^{(m-1)}\|^2$$
$$\widehat{S}^{(m)} = \arg \min_S \|X - \widehat{C}^{(m)}S\|^2$$

under suitable constraints (nonnegativity, coefficients sum...).

Results

Accuracy: mean error for the coefficient matrix around 3%.